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THE BUNDLING HYPOTHESIS

How Perception and Culture Give Rise to Abstract Mathematical Concepts in Individuals

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The "bundling hypothesis" describes the development of abstract mathematical concepts through learning. We present its elements through the investigation of a single conceptual change using multiple methodologies ranging from functional magnetic resonance imaging (fMRI) to novel classroom instruction. The conceptual change of interest is the transition from natural numbers to integers, which further include zero and the negative numbers. The transition is a non-destructive conceptual change. It does not require "a radical reorganization of what is already known" about natural numbers (Stafylidou & Vosniadou, 2004, p. 504). Yet it is still a strong instance of a conceptual change, because the integers cannot be derived from the natural numbers. They depend on the additional mathematical structure of the additive inverse: $X + -X = 0$. For the integers, people need to realize a fundamentally new structure within their concept of number.

The integers present an additional conceptual challenge for learners. Negative numbers do not have a "natural" perceptual referent. In this sense, the integers are abstract. One does not hold negative objects in one's hand, and zero is arguably the prototype of abstractness – structure without substance. Nevertheless, our proposal is that people represent the increased structure of the integers by bundling in new perceptual-motor functionality not found for the natural numbers. In short, people recruit symmetry to embody the additive inverse in their representation of the integers. Without integrating symmetry into their integer representation, people can still solve integer problems by rule, but their understanding is neither deep nor flexible.

Figure 17.1 provides a schematic of the bundling hypothesis when applied to the natural numbers and the integers. The hypothesis is derivative of Case and colleagues' (Case et al., 1997) argument that a rich understanding of number depends on the integration of otherwise separate representations into a core conceptual structure. Our

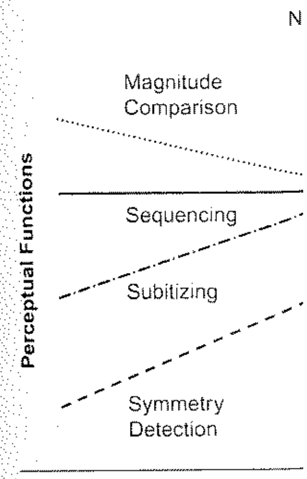


Figure 17.1 The bundling hypothe

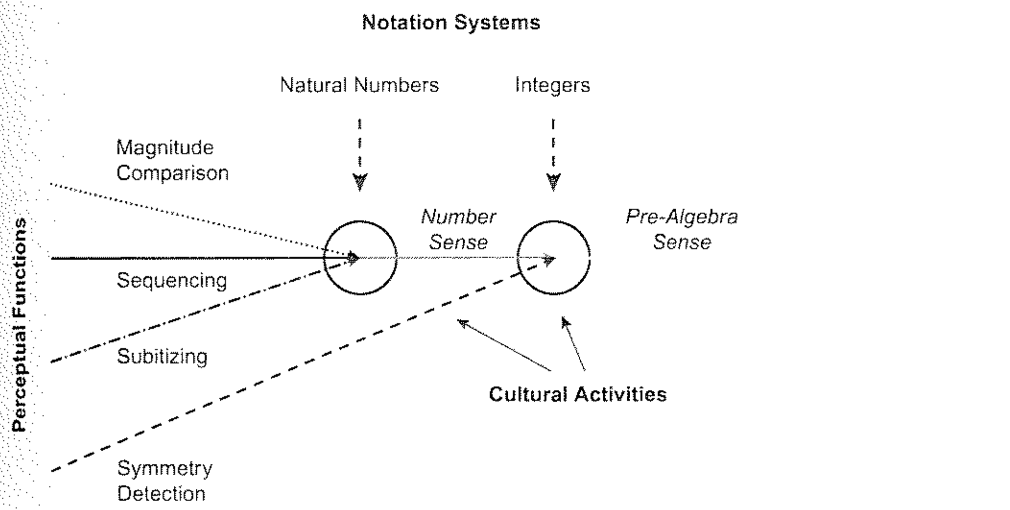


Figure 17.1 The bundling hypothesis

basic tenet is that people have perceptual-motor functions that exist prior to a full understanding of number and that are found in infants and animals. Conceptual change occurs through the integration of these functionalities, which bring new structure and possible operations to a concept.

A second tenet is that the integration is enabled by notation systems that provide a common external representation that anchors different meanings and perceptual functions (e.g., Goldstone, Landy, & Son, 2010). Notation systems also have syntactic rules that support the formal manipulation of quantities. During manipulations of the quantities, the formal rules ensure the coordination of distinct perceptually based properties, so that a change to one perceivable property is associated with changes in another. Cultural activities, such as explicit instruction, help learners notice and bind the appropriate perceptual functions through the notations to create an integrated internal representation.

For the natural numbers, Case et al. (1997) hypothesized that number sense depends on integrating different quantitative competencies that appear separately in infants as basic perceptual-motor schemes. For example, infants can discriminate the magnitudes of physical stimuli (sound, size); they can sequence their own physical movements (motor plans); and they can distinguish small discrete amounts without enumerating (subitizing). Separately, these basic schemes enable the quantitative properties of magnitude, ordinality, and cardinality, respectively. According to the bundling hypothesis, these discrete perceptual-motor uses of quantitative information are integrated through the symbolic, notational structures of mathematics. For example, the digit 5 can refer to the magnitude of a sound (five decibels); it can refer to the order of a sound (fifth); and, it can refer to the number of sounds (five taps). The notation system permits the different meanings to make contact, and articulated symbol systems have their own sets of rules that enforce the coordination for how changes to one meaning affect other meanings. For instance, adding 1 to a set of 5 (a change in cardinality) also increases the bigness of the set from 5 to 6 (a change in magnitude). Appropriate instructional activities can help people learn to coordinate and integrate the different aspects of quantity. In teaching studies, Case and colleagues found that instruction that integrates separate quantitative meanings is more effective than instruction that strengthens each meaning separately (Griffin, Case, & Siegler, 1994).

The bundling hypothesis is consistent with the basic claims of diverse developmental theories. It agrees with nativist arguments that the relevant structure finds its basis in capacities conferred by evolution (e.g., Chomsky, 1966; Spelke, 2000). It also agrees with embodied arguments that the basic building blocks of abstract understanding begin in perception and possible physical actions in the world (e.g., Glenberg & Kaschak, 2002; Lakoff & Núñez, 2000). Finally, it reconciles these tenets with the constructivist tradition that argues representations are constructed or assembled through experience (Piaget, 1952). It does this by proposing that native abilities need to be integrated to make a well-rounded concept of number. The integration allows new concepts to emerge that are more than the sum of their component parts. For example, an understanding of the natural numbers allows for more precision and flexibility in interacting with quantities than would an innate approximate magnitude system.

For the natural numbers, one could argue that the "built in" capacities of humans for perception and construction are sufficient to explain development. Even monkeys can learn to associate natural number symbols with the perceptual referents (Cantlon & Brannon, 2007), and humans may be evolutionarily hardwired for natural numerical processing in much the same way as has been argued for language processing. In contrast, the integers are a relatively new and abstract cultural innovation, with their first full expression occurring only a few hundred years ago. There is no blueprint for integer concepts in the unformed child, and there is no maturation plan for the emergence of integers. Instead, people need to exapt abilities that evolved for one type of problem to help with another. At the cortical level, Dehaene and Cohen (2007) have called this the neuronal recycling hypothesis. Brain regions that are good at specific types of computations are repurposed so they can handle (and enable) cultural demands that rely on those computations. As we present below, brain regions that support the detection of symmetry may be recycled to help with integers.

Culture provides the resources and pressures to help integrate functionality in specific ways. There are two research traditions of special importance. The first involves the abovementioned role of inscriptions and symbolic rules for organizing thought (e.g., Cole & Engeström, 1993; Lehrer & Schauble, 2000). The second involves the influence of sociocultural processes for driving specific forms of cognitive reorganization (e.g., Saxe, 1981, 1988). Vygotsky (1986), for example, proposed that culture-level "scientific" ideas help drive development through a process of internalization from the social plane to the individual plane. As discussed by Leach and Scott (2008), internalization does not mean "direct transfer"; rather the individual interprets the ideas encountered on the social plane. Quoting Vygotsky's contemporary A. N. Leontiev, "the process of internalization is not the transferral of an external activity to a pre-existing 'internal plane of consciousness': it is the process in which that plane is formed" (cited in Leach & Scott, 2008, p. 655). To understand this process, it is important to understand the building blocks that enable and constrain learning. The bundling hypothesis is an attempt to integrate the insights of developmental and sociocultural traditions to explain conceptual change.

In the case of the integers, we claim that basic perceptual-motor capacities for symmetry become bundled together with other quantitative properties through the influence of cultural inscriptions and social interactions. To support this hypothesis, we make an argument with three steps. The first step is to show that educated adults exhibit symmetry-enabled processes when reasoning about integers. This helps to demonstrate that people's understanding of the abstract integers is indeed grounded in perceptual-motor functionality. To simplify subsequent exposition, we use the term "analog" to refer to representations and processes that borrow their structure from perceptual-motor activity (Shepard & Cooper, 1986). Analog means continuous per physical experience, and it should be distinguished from syntactic or verbal rules.

The second step is to show that symmetry has been bundled with other quantitative properties of number, namely magnitude. We also show that adults have developed an analog representation of negative number magnitudes in their own right. In contrast, we find that children who have learned the integers appear to reason by rule rather than using an analog representation. This leads to the final step of our argument. Current curricula for teaching children about integers do not incorporate symmetry, which may explain why the children had not bundled symmetry into their understanding of the integers. (The adults may have learned to integrate symmetry during algebra, which requires balancing equations and interpreting quadrants in Cartesian coordinates.) Therefore, we conducted an instructional study. One condition emphasized symmetry. The students in the symmetry condition were able to solve a greater array of problems compared to students who learned in more traditional ways. This supports the claim that cultural activity, in this case instruction, helps bundle together the perceptual functions into a coordinated representation.

EVIDENCE OF SYMMETRY IN THE INTEGER REPRESENTATION

The first step in our argument involves evidence of analog representations when reasoning about integers. We begin this section with behavioral data and end with brain data. When looking for behavioral evidence of analog representations in cognitive phenomena, researchers often examine overt motor behavior. Examples include the spontaneous use of gestures while problem solving (Schwartz & Black, 1996) and the facilitation or interference of inferences by enforced gestures (Schwartz & Holton, 2000). In most cases, these studies use tasks that require some form of spatial information to achieve an answer. For example, participants may be asked about the spatial behavior of a mechanical system (Hegarty, 1992), whether a tomato can be squeezed (Klatzky, Pellegrino, McCloskey, & Doherty, 1989), or the direction of object motion (Wexler & Klam, 2001). Here, we take a different approach, because we want to show that even abstract problems can involve analog representations.

We set a number of constraints to ensure compelling evidence. First, we wanted evidence of analog representations using a task that does not display relevant spatial information or require spatial manipulation. If successful, it would constitute strong evidence of analog representations in abstract reasoning (as opposed to perceptual involvement in a perception-like task). Second, we did not want to rely on evidence of overt motoric movement, which can often be discounted as a correlate of problem solving rather than a cause. Instead, we looked for response time profiles that implicate analog representations. This methodology is characteristic of research on analog imagery (Shepard & Cooper, 1986), where people exhibit response times that indicate they are mentally rotating imagined objects.

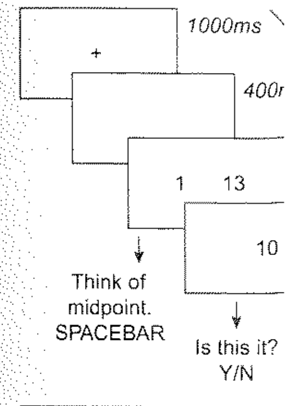
One example of a relevant paradigm that fits these requirements comes from research on the comparison of quantitative magnitudes. In a seminal study, Moyer and Landauer (1967) had participants judge which of two natural number digits was greater (or lesser). Participants exhibited a *symbolic distance effect*: They were faster comparing digits that were quantitatively far apart (1 vs. 9) than near together (1 vs. 3). It is important to note that the digits were not further apart on the screen, so it was the implied magnitude differences that drove the results, not spatial stimuli. The symbolic distance effect is commonly interpreted as evidence for a mental number line such that magnitudes that are farther apart on the line are faster to discriminate (Restle, 1970). A related finding is the *size effect*: For pairs of natural numbers of equal separation, people are faster comparing smaller numbers (1 vs. 4) than larger numbers (6 vs. 9) (Parkman, 1971). The size effect indicates that the number line is psychophysically scaled, because smaller numbers are easier to discriminate than larger numbers. In sum, the time it takes to judge which of two positive number digits represents a greater magnitude exhibits a continuous logarithmic function. Importantly, the distance and size effects are also found when people compare physical quantities such as the loudness of two tones. Brain data indicate that a common brain region is involved in both number magnitude comparisons and physical stimuli comparisons. The intraparietal sulcus (IPS) shows parametric modulation; it is more active for the harder near-magnitude comparisons than for the easier far-magnitude comparisons whether the stimuli are digits or physical magnitudes (Ansari, Garcia, Lucas, Hamon, & Dhital, 2005; Gobel, Johansen-Berg, Behrens, & Rushworth, 2004; Kaufmann et al., 2005; Pinel, Dehaene, Riviere, & LeBihan, 2001).

We adopted the methodological logic of the symbolic distance effect to examine whether people rely on symmetry to reason about integer problems. In our task, people saw a pair of symbolic digits, and we asked them to find the quantitative mid-point of the digits. This bisection task can be solved without imagining a number line, for example, by adding the two digits and dividing them by 2. If people exhibit evidence of symmetry in this task, it makes a strong case that people use analog representations for the abstract integers.

We predicted that people would be faster for bisection problems when the two digits were more symmetric around zero (if put on a number line). For example, people should be faster to find the mid-point of –4 and 6 compared to –2 and 8, because –4 and 6 are more symmetric with respect to zero. This would correspond to findings using visual displays that exhibit various degrees of spatial symmetry. Royer (1981) found that people are faster to judge the symmetry of a visual display as the display exhibits stronger symmetry.

Tsang and Schwartz (2009) asked adults to solve a series of bisection problems. The left side of Figure 17.2 shows the basic task, and the right side shows examples of the types of problems people received. There were perfectly symmetric problems (6 and –6) and perfectly anchored problems (0 and 12). Badland problems were as far away from either as possible (4 and –8), and nearly symmetric and nearly anchored problems were somewhere in between. People also received pure positive problems and pure negative problems that did not cross the zero boundary.

Figure 17.3 shows how long it took people to answer the bisection problems. When problems were perfectly symmetric or anchored, people were very fast, presumably because these were well-memorized number facts. Of more interest are the "tuning" curves. People became progressively faster as the digits neared quantitative symmetry



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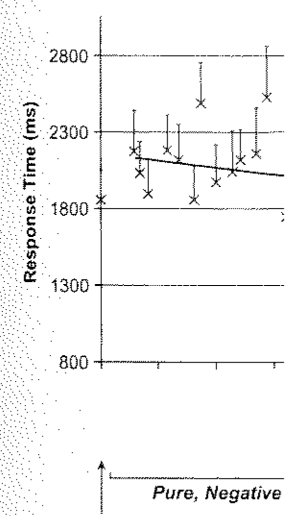

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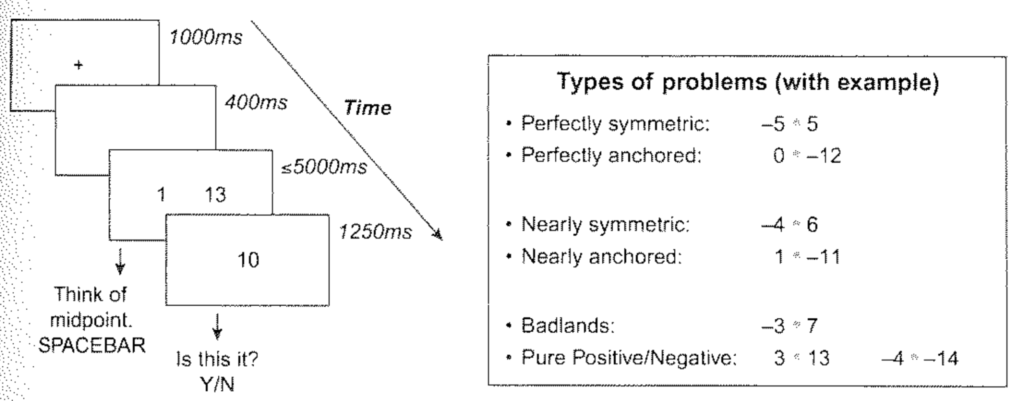

Figure 17.2 Behavioral task for detecting if people recruit symmetry for a purely symbolic task

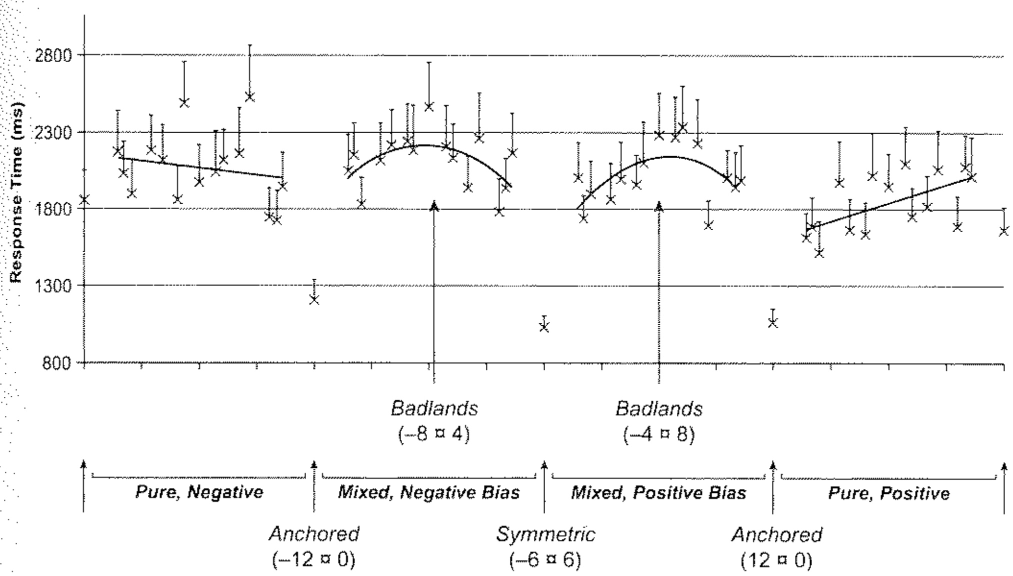

Figure 17.3 People bisect two digits faster when the digits approximate the additive inverse (symmetric) or additive identity (anchored on zero) (adapted from Tsang & Schwartz, 2009)

about zero. Although the task was purely symbolic and the digits always appeared in the same display locations, people seemed to be taking advantage of the implied quantitative symmetry. These results held whether people saw the target digits one after another instead of side-by-side, and whether they were asked to approximate the answer or told to use a formula to find the answer: $(a + b)/2$. The fact that the pattern appeared even when people used a symbolic algorithm indicates that analog symmetry is built deeply into the integer representation.

A limitation of these data is that people also responded faster as problems became more anchored (one of the digits was a neighbor of zero). Was the same underlying process responsible for the improved performance for the symmetric and anchored sides of the curve, or was the symmetry performance due to symmetry specific processes? This is where neural evidence can be useful. It may show differences in underlying process despite behavioral similarities.

Tsang, Rosenberg-Lee, Blair, Schwartz, & Menon (2010) conducted the same study in an fMRI paradigm. Figure 17.4 shows three regions that increased activity as problems became more symmetric and responses became faster. These three regions did not show increased activity as problems became more anchored, despite similar reductions in response time. Thus, the activation was not a general side-effect of faster response times, but rather, it was selective for the implied quantitative symmetry of the stimuli. The involved brain regions have also been implicated in perceptual processes involving symmetry. The left inferior LO (lateral occipital) cortex is implicated in the perception of visually symmetric stimuli (Sasaki, Vanduffel, Knutsen, Tyler, & Tootell, 2005; Tyler et al., 2005) and regions close to the right MTG (middle temporal gyrus) and nearby superior temporal regions are implicated in visual bisection tasks (de Schotten et al., 2005; Wilkinson & Halligan, 2003).

In summary, given a display of purely symbolic stimuli exhibiting no relevant spatial information (i.e., digits), people demonstrated a tuning curve that indicates sensitivity to implied symmetry. People were faster to bisect problems that were near symmetric with respect to zero (−5 and 7). Additionally, brain regions associated with perceptual symmetry became parametrically more active as problem pairs approached symmetric. Pending further evidence, it appears that people have developed internal representations of the integers that embody the integers' new structure – the additive inverse – in symmetry.

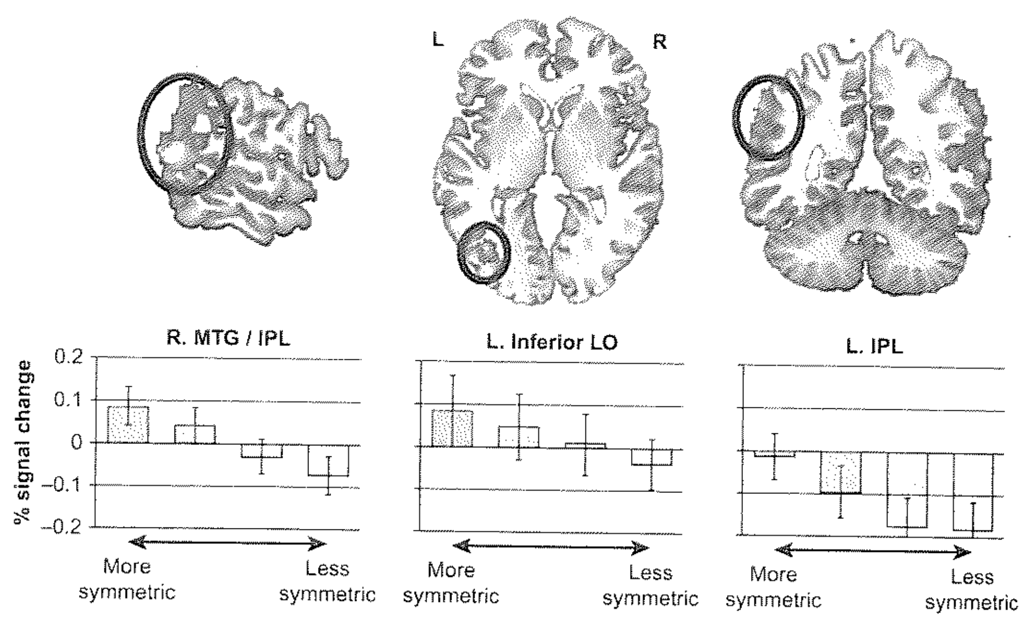

Figure 17.4 Areas of the brain that increased activation as bisection problems increased in symmetry about zero. The top panel shows the location in the brain of three areas that increased activation as the implied symmetry of the digit pairs increased (i.e., the degree to which the two digits approximated the additive inverse). The bottom panel plots the activation changes as the digit pairs became more and less symmetric. All areas of the brain are constantly active, and percentage signal change refers to the change in activity over a baseline level (adapted from Schwartz, Blair, & Tsang, 2012).

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ANALOG MAGNITUDE OF THE NEGATIVE NUMBERS

The preceding section argued that people rely on symmetry to facilitate thinking about integers. This section argues that symmetry has been bundled with other properties of the integers. It does so by developing evidence of symmetry involvement when people complete a task that requires comparing magnitude rather than bisection. If people exhibit evidence of symmetry in a magnitude comparison task, it implies that symmetry has been bundled up with other properties of the integers such as magnitude.

A second issue addressed by this section is whether people represent the magnitude of negative numbers in their own right. One possibility is that people do not have a distinct analog representation of the negative numbers. Instead, people might reason about negative numbers by using their analog representation of natural number magnitude and adding a few supplementary rules (Fischer, 2003; Ganor-Stern & Tzelgov, 2008; Prather & Alibali, 2008; Shaki & Petrusic, 2005; Tzelgov, Ganor-Stern, & Maymon-Schreiber, 2009). Given the task of deciding whether –2 or –5 has a greater magnitude, people can compare positive 2 and 5, and then give the opposite answer ("5 is larger than 2, so the correct answer is that –2 is larger than –5"). Or, when asked which is the greater of –5 and 2, they can simply rely on the rule that positives are always greater than negatives. In this hybrid representation of negative numbers, the sense of magnitude comes from the positives, and people add new structure by using symbolic rules.

A second possibility is that people might represent negative magnitudes in their own right by developing a "leftward" extension of the conventional number line. Using this extended model, people could compare –2 and –5 directly.

Varma and Schwartz (2011) found that neither the first nor the second possibility is exactly right. While educated adults can reason about negative numbers by using positive numbers supplemented with rules, they also appear to have a representation of negative numbers in their own right that they can call upon, and this representation is symmetric to the positives rather than a linear extension. In contrast, 12-year-old children who are relatively new to negative numbers appear to use a hybrid model.

Varma and Schwartz asked adults and 6th-grade children to make speeded judgments about which of two digits referred to a greater amount. People saw two digits on a screen, and they had to press a button on the side of the greater digit. (For this task, participants understood that a –2 should be considered a greater amount than –5. In its own right, this is an important development, because children need to uncouple magnitude and direction for the integers; Bofferding, 2011.) People completed three major types of comparison problems: pure positive problems (e.g., 5 vs. 2), pure negative problems (e.g., –5 vs. –2), and mixed problems that spanned the zero boundary (e.g., –5 and 2).

The key evidence for an analog representation of magnitude comes from the comparison of response times for near and far problems. Near problems involved digit pairs that were three or fewer steps apart (e.g., 1 vs. 3, –5 vs. –7, –2 vs. 1). Far problems involved digits that were seven or more steps apart (e.g., 1 vs. 8, –2 vs. –9, –3 vs. 5). If people rely on an analog representation, then near comparisons should take longer than the corresponding far comparisons, per the symbolic distance effect described earlier. Again, the logic is that similar magnitudes (near) should be harder to discriminate than more distinct magnitudes (far).

Adult Representations of Negative Numbers

Figure 17.5 shows the results. We first consider the adult pattern. Both the pure positive and pure negative problems show the signature of an analog magnitude representation – near comparisons take longer than far comparisons. The striking finding involves the mixed comparisons, which showed an *inverse* distance effect (cf. Krajcsi & Igács, 2010). People were faster for near-mixed comparisons than for far-mixed comparisons. For example, people were faster to judge the larger of –1 vs. 2 than –1 vs. 7. By a pure magnitude account, the former comparison should be harder, not easier, because the digits are closer in magnitude. This suggests the adults have incorporated some additional structure into their representation of integers besides pure magnitude.

Varma and Schwartz (2011) created the mathematical model in Figure 17.6 to account for these distance effects. The model also incorporates the standard size effect: Smaller magnitudes (1 vs. 3) are compared more quickly than larger magnitudes (7 vs. 9) when they are both the same distance apart. This is why the lines are curved logarithmically – smaller numbers are further apart and more distinct in the representation.

The striking characteristic of the best fitting model (Figure 17.6) of the Varma and Schwartz data is that the negatives are a reflection of the positive numbers rather than an extension of the positive numbers. By this model, the analog representation of negative numbers is not simply an internalization of the standard number line. If it were an internalization of the number line, then near-mixed comparisons (1 vs. –2) would be harder, not easier than far-mixed comparisons (1 vs. –7) because the near-mixed comparisons would be closer to each other in the representation. Instead, the representation capitalizes on the symmetry of negative and positives about zero, which makes near-mixed comparisons easier because they are on either side of, and close to, the boundary point of zero.

The visual presentation of the model is not intended to imply that people use a picture of a sideways V when they think about integers. Rather, the model describes the structural

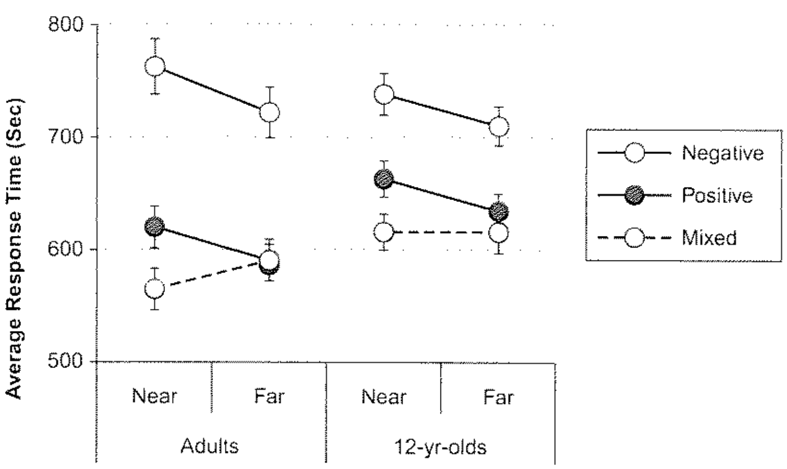
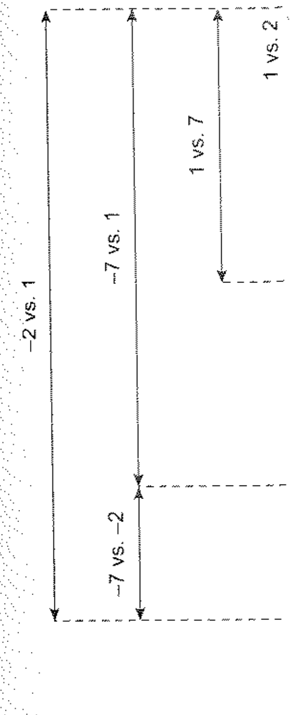


Figure 17.5 Response times for digit comparisons. Positive comparisons involved two positive digits (e.g., 2 vs. 8); negative comparisons involved two negative digits (e.g., –2 vs. –8); and, mixed comparisons involved a positive and a negative digit (e.g., –2 vs. 8). Near comparisons used digits that were within three steps of each other (–1 and –4), and far comparisons used digits that were seven or more steps apart (–1 and –8). (Adapted from Varma & Schwartz, 2011.)



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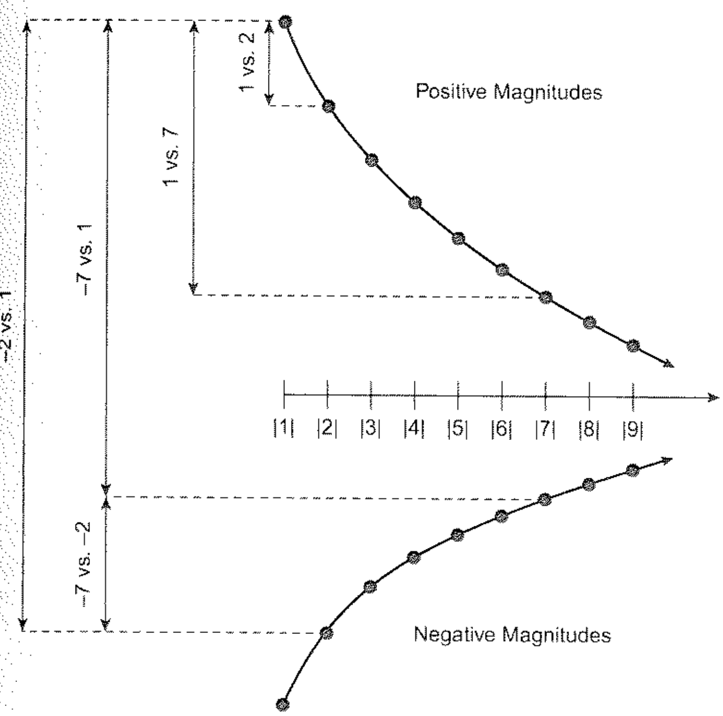

Figure 17.6 Reflection model of integer representation. Positive and negative magnitudes are reflections of one another, which embodies the additive inverse. Magnitude comparisons are a function of the projected distances between numbers, such that −2 vs. 1 is further apart than −7 vs. 1, and therefore should be answered more quickly. Comparisons of negatives versus positives should also be answered more quickly than pure positive or pure negative comparisons. The curvature of the lines indicates that people have superior resolution for comparing small magnitudes compared to large magnitudes, and therefore small magnitudes should be compared faster. (Adapted from Varma & Schwartz, 2011.)

relations that determine what is easy and hard to think about. The model posits that people have bundled the structural relation of symmetry into their analog representation of magnitude.

A second notable point of the model is that the negative magnitudes are less distinct than the positive magnitudes. In Figure 17.4, the negative magnitudes are more compressed than the positive numbers. This makes sense in that people have had much less experience with negative numbers than positive numbers. Blair, Rosenberg-Lee, Tsang, Schwartz, & Menon (2012) examined this effect using fMRI.

Participants performed a similar magnitude comparison task in an MRI scanner. Two numbers were presented on the screen and participants determined the greater or lesser number. Of particular interest was the representation of positive and negative numbers. If people have an independent representation of negative numbers that is compressed relative to positives, as predicted by the mathematical model, we would expect negative comparisons to elicit different activation in the IPS. Recall that the IPS is involved in magnitude processing.

The key analysis examined patterns of activation in the IPS for positive-only and negative-only comparisons independent of overall reaction time or activation level. A representational similarity analysis compared the spatial patterns of activation between

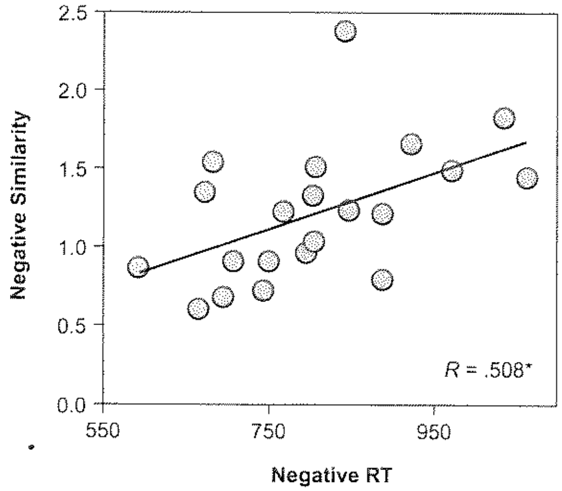

Figure 17.7 Correlation between the degree of neural overlap for near and far negative number comparisons and reaction time for solving negative comparison problems. People who exhibited larger overlap in brain regions for near and far negative comparisons were also slower to answer negative comparison problems.

near and far comparisons for positive numbers and for negative numbers. The logic is that if there is high overlap in regions of activation for the near and far problems in the magnitude processing region (IPS), then people do not have a well-differentiated magnitude representation of the numbers. In contrast, if there is less overlap in regions of activation, then people have a more differentiated representation of number magnitudes. The analysis revealed that neural responses in the IPS were more differentiated among positive numbers than among negative numbers. This helps to explain why people are slower comparing negative numbers – their representation of negative number magnitudes is less well-defined. Figure 17.7 shows that degree of neural overlap for near- and far-negative comparisons is strongly correlated with slower response times for solving negative comparison problems. These findings point to a unique, but less well-developed, magnitude representation for negative numbers, as predicted by the computational model. If individuals converted negative numbers to positive values before making magnitude comparisons there should not have been differences in the similarity for negative problems compared to positive problems, because both comparisons would involve the representation of positive magnitudes.

Immature Representations of Negative Numbers

Next, we return to the behavioral data and the children's results in Figure 17.5. The children were equally accurate as the adults (>95%). However, they show interesting differences in their response time patterns. Most notably, they do not show a distance effect for the mixed comparisons; the response times for near- and far-mixed problems are the same. Rather than consulting a magnitude representation of negative numbers, they were simply using a rule that a positive is always greater than a negative (i.e., they were only looking at the sign on the digits, and disregarding the magnitudes).

A more subtle difference is that the children were faster at making pure negative comparisons than the adults, even though they were slower for the pure positive comparisons. One interpretation of this finding is that the children were solving the pure negative comparison problems by using their well-developed natural number representation and applying a rule to flip the answer. In contrast, the adults relied on their analog representation of negative numbers. Because the adult representation of negative numbers has less resolution than the positive numbers, it took them longer than the children.

In summary, well-educated adults appear to have an independent analog representation of the negative numbers. Moreover, the overall integer representation appears to have bundled magnitude and symmetry together, with negative and positive numbers organized as a reflection of one another. The representation is not simply a copy of the standard number line seen in textbooks, but rather it takes a symmetric form that enhances the additive inverse. This representation embodies the new mathematical structure of the integer system compared to the natural numbers, but the embodiment is not a simple copy of perceptual-motor experience.

The second key finding from this study is that children were able to solve the integer problems, but they did not appear to have a distinct representation of negative numbers. Instead, they were applying symbolic rules to augment their natural number representation. This yields two points. The first is the well-known observation that people can solve mathematical problems by reference to symbolic rules and math facts without using an underlying semantic representation. This can yield much faster problem solving but, as we demonstrate below, it also leads to less flexibility and generativity. The second point involves the cross-sectional comparison of the children and the adults. The children solved the problems using an abstract rule (e.g., a positive is always greater than a negative) without reference to a representation of negative numbers. In contrast, the adults solved the problems by reference to a more perceptually derived representation. This reverses the usual concrete-to-abstract learning progression (Bruner, 1996; Piaget, 1941). With the integers, there appears to be an abstract-to-concrete shift.

Per the bundling hypothesis, we speculate that over time the symbolic representation and structure of the integers slowly enlists perceptual-motor representations. When learning integers, children already have an analog magnitude representation of natural number that exhibits perceptual-motor properties (Sekuler & Mierkiewicz, 1977). Because negative numbers and zero do not have a ready perceptual-motor basis, children initially understand them by using symbolic rules that map them to natural numbers. Over experience, the structure of the syntactic rules and their operations over integers changes the original magnitude representation of natural number to directly embody the unique properties of the integers, such as the fact that zero is a boundary between positives and negatives. By the bundling hypothesis, appropriate engagement with symbolic notations provides a mechanism for transforming perceptual-motor experiences into richer, abstract concepts.

CULTURAL SUPPORTS FOR BUNDLING SYMMETRY INTO THE INTEGERS

The first step in our support of the bundling hypothesis was evidence that people use symmetry to think about purely symbolic problems involving integers. The second step was to show that symmetry has been bundled into the other properties of number, specifically magnitude. The third step is to show that cultural organizations of representations and activities can facilitate the bundling of symmetry into the integers. To complete this theoretical demonstration, we turned our basic hypothesis and research into a practical application that improves early instruction involving the integers.

Instruction that introduces the integers usually uses a number line model, a cancellation model, or both (Bofferding, 2011; Liebeck, 1990; Gregg & Gregg, 2007). In the number line model, students are introduced to negative numbers as a leftward extension of the natural number line. Addition and subtraction with integers is modeled as movement along the number line. For example, to do addition in enVisionMATH (www.pearsonschool.com/envisionmath), students imagine standing on the first addend on the number line and facing the positive direction. Walking forward means adding a positive number and walking backward means adding a negative number. For subtraction students face the negative direction and walk forward for subtracting a positive and backward for subtracting a negative. In the number line model, integers can be thought of as positions and arithmetic can be thought of as movement or changes in position.

In the cancellation model students are encouraged to think of integers as amounts rather than distances and directions, with the negative and positive integers representing opposite quantities. Students learn that the positive and negative quantities cancel each other out, and they model arithmetic problems using a set of concrete counters (small disks). For example, for the addition problem –5 + 2, five negative (sometimes red) counters are placed in a row, two positive (sometimes yellow) counters are placed in a parallel row, and the positive and negative counters that match up cancel each other out, leaving –3 remaining as the answer. For both the number line and cancellation models, teachers may also introduce a set of rules to supplement the physical representation. For instance, given the problem 3 + (–5) students can learn to subtract the smaller number from the larger number ignoring the signs, and attach a negative sign when the original negative addend is larger than the positive addend.

Notably, these curricula do not explicitly incorporate symmetry. To find out if including symmetry would improve student learning, we conducted a study with fourth graders who had not yet learned about negative numbers. Students learned over four days in one of three instructional conditions: Jumping, Stacking, and Folding. The Jumping and Stacking conditions mapped into current instructional models and the Folding condition additionally introduced symmetry. The conditions are named for the core action students engaged in while physically modeling integer addition problems: jumping a figurine along a number line, stacking blocks on a number line, or folding the positive and negative sides of the number line together.

The purpose of the physical activities with the manipulatives was to draw student attention to the key properties of interest (e.g., folding about a symmetry point). Simply exposing students to appropriate cultural representations or phenomena is not sufficient to ensure recruitment of the appropriate mental functionality. People need to learn to notice the relevant property and operations. Any given action or perceptual stimuli has an infinite amount of information (Gibson, 1969), so it is not enough to just assume that people will pick up the relevant properties, even if they are implicit in their actions or perceptions. For example, consider the traditional number line in Figure 17.8. It exhibits a variety of properties such as ordinality and equal intervals. It also exhibits edges, curved fonts, placement on the page, arrows on the ends, and so on. It also includes symmetry

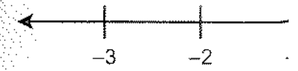

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Figure 17.8 Standard representation of the integers in instruction

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The first phase of the instruction comprised a series of instructional games using physical manipulatives that we designed to draw student attention to different ways of thinking about negatives. Figure 17.9 shows how the conditions differ when playing a simple addition game. The Jumping condition (Figure 17.9C) enacted the addition problems in each game by hopping a plastic figurine back and forth along the number line. The Stacking condition (Figure 17.9B) instead used a representation of stacked blocks. For example, for +3 + −2, students started at zero and placed three blue blocks on the positive side of the number line to represent +3. Then, working their way back toward 0, they placed two red blocks on top to represent −2. The stacked portion "canceled out," and the remaining unstacked portion was the answer, in this case 1 blue positive block. For the Folding condition (Figure 17.9A), the manipulatives included a hinge at the number zero, allowing the students to fold the number line in half. Students represented both addends on the number line at the same time, for example with three blue blocks on the positive side and two red blocks on the negative side. They then snapped the sides together by folding at zero. The blocks that matched up after folding canceled out, and the ones that did not match up were the remaining answer.

After two days of working with the manipulatives, students transitioned to computer games. The computer games maintained the differences between conditions, and they enabled students to complete more questions in the available time (students did not have to fuss with the concrete materials). With the computer games students did not engage in physical behaviors that had any sort of correspondence to manipulating the physical number line objects. Instead, the computer games showed the respective spatial representations and transformations, and students simply entered symbolic answers through discrete clicks. Over time, the computer games faded the spatial representation of the manipulatives so that students were working primarily with symbolic digits.

The study included a number of learning measures. On a simple measure of adding negative and positive digits, all the conditions improved significantly with no differences between conditions. This indicates that students learned the basic content from all three conditions. The post-test also included questions that required students to extend their understanding to new problems that they had not been taught, such as placing positive and negative fractional amounts on the number line or solving missing variable problems with integers. On these generalization items, students in the folding condition performed significantly better, indicating that they developed a more flexible understanding of integers (Figure 17.10).

Additionally, instruction influenced children's strategies for solving problems. Students in the folding condition used significantly more symmetry-based strategies, which were associated with better performance. For example, when asked to put the number 4 on a number line on which only −4 and 0 were marked, students in the symmetry condition were more likely to measure the distance from 0 to −4 with their fingers, and then measure

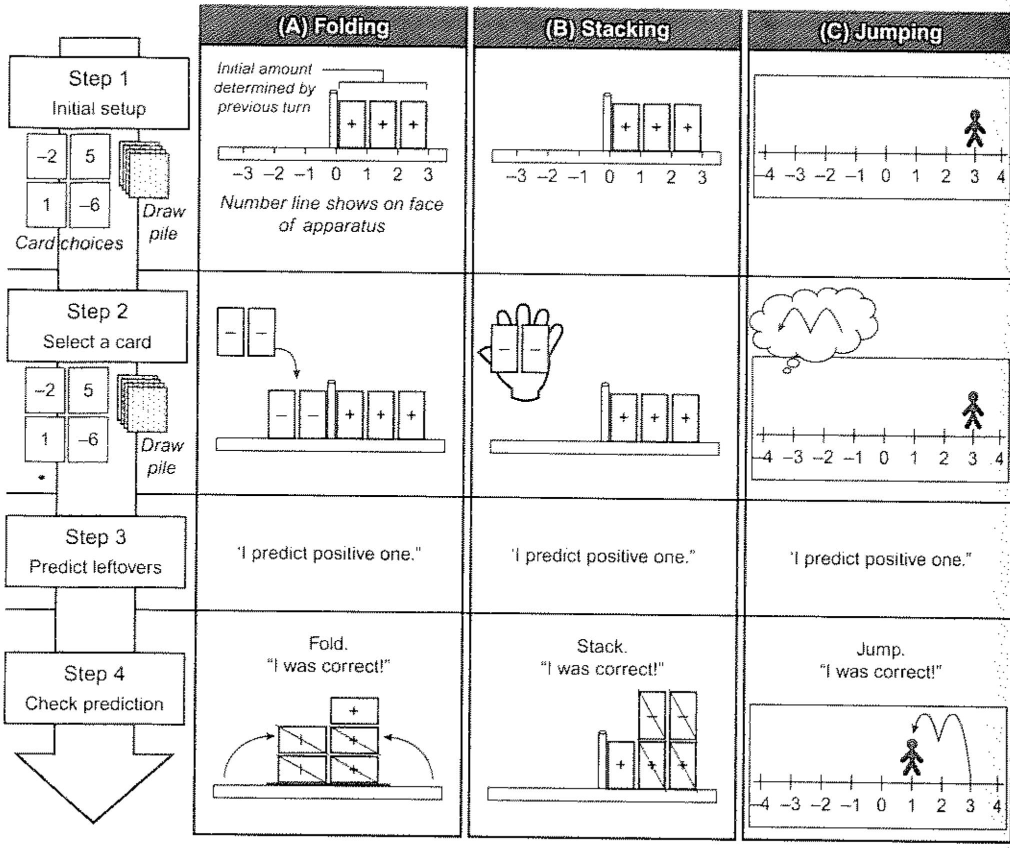
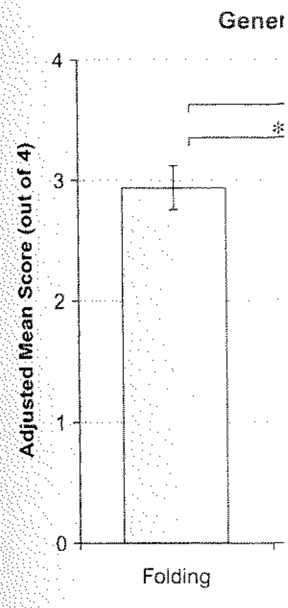


Figure 17.9 Leftovers Game schematic for each instructional condition. This is a 2–4 player card game played during the second half of the curriculum. Left column shows the steps for one turn in the game. The three panels show the corresponding actions with manipulates for the (A) Folding, (B) Stacking, and (C) Jumping conditions. The example shown is equivalent to solving the equation 3 + −2 = 1 (adapted from Tsang, 2012).

the same distance on the right side of 0 to place positive 4. Students in the other conditions were more likely to start at 0 and draw four tick marks in the positive direction, placing the number 4 at the fourth tick mark.

Tsang (2012) also included a measure that looked for negative side-effects of the symmetry instruction. As people develop structures that are optimized for handling classes of problems, these structures may interfere with other classes of problems that would be better served by different organizations of knowledge. Evidence for this point comes from a computerized reaction time measure. A number line was shown on the screen with only the endpoints labeled. For some trials, the endpoints were symmetric (e.g., −6, 6), such that 0 would fall in the center of the line. For other trials, the endpoints were non-symmetric (e.g., −4, 8) such that zero would not be in the center of the line. One part of the number line was occluded by a green box. Students indicated whether the number 0 would fall in the area occluded by the box. Students in the jumping and stacking conditions were unaffected by whether the endpoints on the number line indicated symmetry or not. In contrast, students in the folding condition were significantly slower for the non-symmetric problems. These students had learned to rely on



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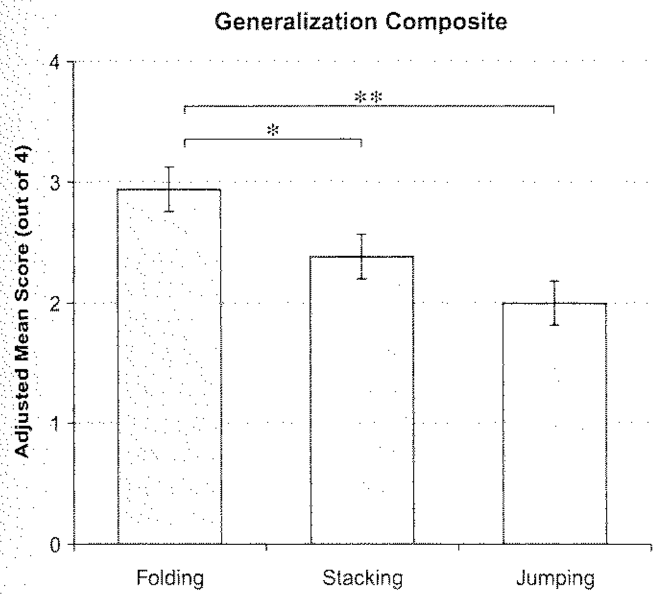

Figure 17.10 Score on generalization composite by condition (adapted from Tsang, 2012)

symmetry, and when it was not there, they had to refigure another way to solve the problem.

Ideally, people develop appropriate structure for the most prevalent and important classes of problems they will experience, and situations where that structure is an impediment are rare. This appears to be the case for symmetry and the integers. Regardless of condition, students who exhibited greater interference for the non-symmetric problems did much better on the generalization composite score that required solving novel quantitative problems. They had developed a better representation of integer symmetry, which helped them with most problems, but also hurt them on problems designed to make a symmetry orientation sub-optimal. Combined, these results indicate that students in the symmetry condition were more likely to incorporate the relationship of symmetry into their representations of integers, as evidenced by reaction time, and across conditions, students whose representations incorporated symmetry had a more flexible and generalizable understanding of the integers.

CONCLUSION

The bundling hypothesis attempts to provide a mechanistic account of how new mathematical concepts are formed. Through socially organized interaction and highly structured cultural symbol systems, distinct perceptual-motor functions get bundled together to enable new concepts with a higher-order structure. We provided a chain of evidence regarding integers that supports several of the claims within the bundling hypothesis. Adults exhibit behavior and brain activation that indicates the involvement of symmetry when finding the midpoint of a positive and negative digit. Symmetry appears to be bundling into adult sense of magnitude, because they show distinct patterns of response times when comparing the magnitudes of digits on either size of zero. Adults also exhibit a distinct neural representation of negative numbers, and adults who have more spatially distinct brain patterns of activation for negative number comparisons also show faster response times. Children who have had traditional instruction do not exhibit these patterns, but instead, they appear to rely on rules that allow them to solve problems by consulting their representation of positive numbers supplemented by rules. However, with instruction that emphasizes symmetry, children exhibit evidence of relying on symmetry when they solve visual bisection tasks. Moreover, students who have integrated symmetry into their understanding of integers are more successful at solving problems that go beyond what they have been directly taught.

There are aspects of the bundling hypothesis that were not investigated. We did not seek direct evidence on the importance of instruction that integrates the different perceptual properties into a common external representation. We have suggestive evidence but it has not been rigorously tested. In pilot work, we taught children about symmetry without making careful connections to the other properties of number (interval, magnitude, etc.). Symmetry turned into a "free-floating" property of the integers. For instance, the children never noticed that positives go on the right and negatives go on the left. We also have not conducted a pre–post intervention where we examine whether children increase the recruitment of brain regions associated with symmetry after instruction, and thus, we have not closed the loop showing that instruction causes "bundling" within the brain. We have not determined whether the involvement of symmetry makes a difference as students move to more complex topics such as algebra, which makes great use of the additive inverse.

In the meantime, if we assume that something close to the bundling hypothesis is true for the integers, are there other conceptual changes where the hypothesis would be a possible explanation? Would the bundling hypothesis help explain how people manage to find meaning and structure in other highly abstract mathematical concepts such as calculus's "instantaneous change"? Perhaps the bundling hypothesis can provide a good account of children's transition from the interval scale of natural numbers to the ratio scale of rational numbers. In addition to overcoming a simple natural number interpretation of fraction notations (e.g., Stafylidou & Vosniadou, 2004), children need to recruit a "sense of ratio." According to the bundling hypothesis, this would be found in native perceptual abilities that can provide the additional structure for rational number. There are many possible candidates, given that visual structure is often invariant of absolute size.

The bundling hypothesis is an attempt to explain how new structure arises in the mind of the learner. It is not an account of how people jump from one explanatory structure to another during a process of conceptual change, as would be the case in switching explanatory paradigms from mechanical causality to stochastic emergence (Chi, 2005). Rather, it is an attempt to answer the question of how an explanatory structure can develop in the first place. For the integers at least, it appears that culture and external representations help people bind previously discrete perceptual-motor systems into integrated conceptual structures.

Ansari, D., Garcia, N., Luc
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Bofferding, L. C. (2011).
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Bruner, J. (1996). *The cult*
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Case, R., Okamoto, Y., G
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Development, 61, 1–2
Chi, M. T. H. (2005). Co
Journal of the Learnin
Chomsky, N. (1966). *Car*
Row.
Cole, M., & Engestrom, Y
Distributed cognition
Dehaene, S., & Cohen, L.
de Schotten, M. T., Urba
parietal-frontal path
Fischer, M. H. (2003). Co
Ganor-Stern, D., & Tzelgo
157–163.
Gibson, E. (1969). *Princip*
Glenberg, A. M., & Kasc
558–565.
Gobel, S. M., Johansen-B
activation during nu
Goldstone, R. L., Landy, D
Gregg, J., & Gregg, D. U.
13(1), 46–50.
Griffin, S. A., Case, R., &
formal learning of a
Integrating cognitive t
Hegarty, M. (1992). Men
Experimental Psychol
Kaufmann, L., Koppelsta
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NeuroImage, 25(3), 8
Klatzky, R. L., Pellegrino, J
representations in se
Krajcsi, A., & Igács, J. (201
shortcut. *European Jo*
Lakoff, G., & Núñez, R. (
being. New York, NY:
Leach, T., & Scott, P. H. (
sociocultural perspec
Routledge.
Lehrer, R., & Schauble, L
students' classificatio
Liebeck, P. (1990). Scor
Mathematics, 21(3),
Moyer, R. S., & Landaue
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REFERENCES

Ansari, D., Garcia, N., Lucas, E., Hamon, K., & Dhital, B. (2005). Neural correlates of symbolic number processing in children and adults. *NeuroReport, 16*(16), 1769–1773.

Blair, K. P., Rosenberg-Lee, M., Tsang, J. M., Schwartz, D. L., & Menon, V. (2012). Beyond natural numbers: Negative number representation in parietal cortex. *Frontiers in Human Neuroscience, 6*(7), 1–17.

Bofferding, L. C. (2011). *Expanding the numerical central conceptual structure: First graders' understanding of integers.* Doctoral dissertation, Stanford University, Stanford, CA.

Bruner, J. (1996). *The culture of education.* Cambridge, MA: Harvard University Press.

Cantlon, J. F., & Brannon, E. M. (2007). Basic math in monkeys and college students. *PLoS Biology, 5*(12).

Case, R., Okamoto, Y., Griffin, S., McKeough, A., Bleiker, C., Henderson, B., et al. (1997). The role of central conceptual structures in the development of children's thought. *Monographs of the Society for Research in Child Development, 61,* 1–295.

Chi, M. T. H. (2005). Common sense conceptions of emergent processes: Why some misconceptions are robust. *Journal of the Learning Sciences, 14,* 161–199.

Chomsky, N. (1966). *Cartesian linguistics: A chapter in the history of rationalist thought.* New York, NY: Harper & Row.

Cole, M., & Engeström, Y. (1993). A cultural–historical approach to distributed cognition. In G. Salomon (Ed.), *Distributed cognitions.* Cambridge, UK: Cambridge University Press.

Dehaene, S., & Cohen, L. (2007). Cultural recycling of cortical maps. *Neuron, 56*(2), 384–398.

de Schotten, M. T., Urbanski, M., Duffau, H., Volle, E., Levy, R., Dubois, B., et al. (2005). Direct evidence for a parietal-frontal pathway subserving spatial awareness in humans. *Science, 309*(5744), 2226.

Fischer, M. H. (2003). Cognitive representation of negative numbers. *Psychological Science, 14*(3), 278–282.

Ganor-Stern, D., & Tzelgov, J. (2008). Negative numbers are generated in the mind. *Experimental Psychology, 55*(3), 157–163.

Gibson, E. (1969). *Principles of perceptual learning and development.* New York: Appleton-Century-Crofts.

Glenberg, A. M., & Kaschak, M. P. (2002). Grounding language in action. *Psychonomic Bulletin & Review, 9,* 558–565.

Gobel, S. M., Johansen-Berg, H., Behrens, T., & Rushworth, M. F. (2004). Response-selection-related parietal activation during number comparison. *Journal of Cognitive Neuroscience, 16*(9), 1536–1551.

Goldstone, R. L., Landy, D., & Son, J. Y. (2010). The education of perception. *Topics in Cognitive Science,* 2, 265–284.

Gregg, J., & Gregg, D. U. (2007). A context for integer computation. *Mathematics Teaching in the Middle School, 13*(1), 46–50.

Griffin, S. A., Case, R., & Siegler, R. S. (1994). Rightstart: Providing the central conceptual prerequisites for first formal learning of arithmetic to students at risk for school failure. In K. McGilly (Ed.), *Classroom lessons: Integrating cognitive theory and classroom practice* (pp. 25–49). Cambridge, MA: MIT Press.

Hegarty, M. (1992). Mental animation: Inferring motion from static diagrams of mechanical systems. *Journal of Experimental Psychology: Learning, Memory and Cognition, 18,* 1084–1102.

Kaufmann, L., Koppelstaetter, F., Delazer, M., Siedentopf, C., Rhomberg, P., Golaszewski, S., et al. (2005). Neural correlates of distance and congruity effects in a numerical Stroop task: an event-related fMRI study. *NeuroImage, 25*(3), 888–898.

Klatzky, R. L., Pellegrino, J. W., McCloskey, B. P., & Doherty, S. (1989). Can you squeeze a tomato? The role of motor representations in semantic sensibility judgments. *Journal of Memory & Language, 28,* 56–77.

Krajcsi, A., & Igács, J. (2010). Processing negative numbers by transforming negatives to positive range and by sign shortcut. *European Journal of Cognitive Psychology, 22,* 1021–1038.

Lakoff, G., & Núñez, R. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being.* New York, NY: Basic Books.

Leach, T., & Scott, P. H. (2008). Teaching for conceptual understanding: An approach drawing on individual and sociocultural perspectives. In S. Vosniadou (Ed.) *International Handbook of Conceptual Change.* New York, NY: Routledge.

Lehrer, R., & Schauble, L. (2000). Inventing data structures for representational purposes: Elementary grade students' classification models. *Mathematical Thinking and Learning, 2,* 51–74.

Liebeck, P. (1990). Scores and forfeits: An intuitive model for integer arithmetic. *Educational Studies in Mathematics, 21*(3), 221–239.

Moyer, R. S., & Landauer, T. K. (1967). The time required for judgments of numerical inequality. *Nature, 215,* 1519–1520.

Parkman, J. M. (1971). Temporal aspects of digit and letter inequality judgments. *Journal of Experimental Psychology, 91*, 191–205.

Piaget, J. (1941/1952). *The child's conception of number.* London, UK: Routledge and Kegan Paul.

Piaget, J. (1952). *The origins of intelligence in children* (M. Cook., Trans.). New York, NY: International Universities Press.

Pinel, P., Dehaene, S., Riviere, D., & LeBihan, D. (2001). Modulation of parietal activation by semantic distance in a number comparison task *NeuroImage, 14*(5), 1013–1026.

Prather, R. W., & Alibali, M. W. (2008). Understanding and using principles of arithmetic: Operations involving negative numbers. *Cognitive Science, 32*, 445–457.

Restle, F. (1970). Speed of adding and comparing numbers. *Journal of Experimental Psychology, 83*, 274–278.

Royer, F. L. (1981). Detection of symmetry. *Journal of Experimental Psychology: Human Perception and Performance, 7*(6), 1186–1210.

Sasaki, Y., Vanduffel, W., Knutsen, T., Tyler, C., & Tootell, R. (2005). Symmetry activates extrastriate visual cortex in human and nonhuman primates. *Proceedings of the National Academy of Sciences of the United States of America, 102*(8), 3159–3163.

Saxe, G. B. (1981). Body parts as numerals: A developmental analysis of numeration among the Oksapmin in Papua New Guinea. *Child Development, 52*(1), 306–316.

Saxe, G. B. (1988). Candy selling and math learning. *Educational Researcher, 17*(6), 14–21.

Schwartz, D. L., & Black, J. B. (1996). Shuttling between depictive models and abstract rules: Induction and fallback. *Cognitive Science, 20*, 457–497.

Schwartz, D. L., Blair, K. P., & Tsang, J. (2012). How to build educational neuroscience: Two approaches with concrete instances. *British Journal of Educational Psychology Monograph Series, 8*.

Schwartz, D. L. & Holton, D. (2000). Tool use and the effect of action on the imagination. *Journal of Experimental Psychology: Learning, Cognition, and Memory, 26*, 1655–1665.

Sekuler, R., & Mierkiewicz, D. (1977). Children's judgments of numerical inequality. *Child Development, 48*, 630–633.

Shaki, S., & Petrusic, W. M. (2005). On the mental representation of negative numbers: Context-dependent SNARC effects with comparative judgments. *Psychonomic Bulletin & Review, 12*(5), 931–937.

Shepard, R. N., & Cooper, L. A. (Eds.). (1986). Mental images and their transformations. Cambridge, MA: MIT Press.

Spelke, E. S. (2000). Core knowledge. *American Psychologist, 55*, 1233–1243.

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction, 14*, 503–528.

Tsang, J. M. (2012). Learning to see less than nothing: Symmetry in the mental representation of integers. Doctoral dissertation, Stanford University, Stanford, CA.

Tsang, J. M., Rosenberg-Lee, M., Blair, K. P., Schwartz, D. L., & Menon, V. (2010, June). *Near symmetry in a number bisection task yields faster responses and greater occipital activity.* Poster presented at the 16th Annual Meeting of the Organization for Human Brain Mapping, Barcelona, Spain.

Tsang, J. M., & Schwartz, D. L. (2009). Symmetry in the semantic representation of integers. In N. Taatgen & H. van Rijn (Eds.), *Proceedings of the 31st Annual Conference of the Cognitive Science Society* (pp. 323–328). Austin, TX: Cognitive Science Society.

Tzelgov, J., Ganor-Stern, D., & Maymon-Schreiber, K. (2009). The representation of negative numbers: Exploring the effects of mode of processing and notation. *Quarterly Journal of Experimental Psychology, 62*(3), 605–624.

Tyler, C. W., Baseler, H. A., Kontsevich, L. L., Likova, L. T., Wade, A. R., & Wandell, B. A. (2005). Predominantly extra-retinotopic cortical response to pattern symmetry. *NeuroImage, 24*(2), 306–314.

Varma, S., & Schwartz, D. L. (2011). The mental representation of integers: An abstract-to-concrete shift in the understanding of mathematical concepts. *Cognition, 121*, 363–385.

Vygotsky, L. S. (1986). *Thought and language* (A. Kozulin, Trans.). Cambridge, MA: MIT Press.

Wexler, M., & Klam, F. (2001). Movement prediction and movement production. *Journal of Experimental Psychology: Human Perception and Performance, 27*, 48–64.

Wilkinson, D. T., & Halligan, P. W. (2003). Stimulus symmetry affects the bisection of figures but not lines: Evidence from event-related fMRI. *NeuroImage, 20*(3), 1756–1764.